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Mathematical Knowledge and Practice (MKP) Standards

for Prospective FP and IP Teachers

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TEACHING & LEARNING DEVELOPMENT CAPACITY IMPROVEMENT PROGRAMME

HRDC
Maths and Science Standing Committee

Contents

Guiding Principles

These principles underpin the specific knowledge and process standards developed within each of the Primary Mathematics strands of Mathematical Acting and Thinking, Number and Algebra, and Geometry and Measurement.

Prospective primary teachers (PPTs) need to have:

GP1 **Well-connected knowledge of primary school mathematics for oneself, and from the perspective of teaching.**
This entails PPTs being able to solve lower and higher cognitive demand tasks related to primary mathematics. It also entails being able to represent, explain and justify all choices within the problem-solving process, with awareness of alternative problem-solving routes.

GP2 **Knowledge of learners and how they develop mathematical understandings.**
This principle includes understanding of children's innate cognitive resources, how these innate resources could be developed, and the characteristics that children who are competent with work in a particular strand are likely to display. It also includes awareness of learners' socio-cultural and linguistic environments as this is needed to make connections between out-of-school and in-school understandings. Knowledge of how to access children's prior understandings is important for building secure trajectories of understanding.

GP3 **Knowledge of teaching practices that develop learners' mathematical understandings.**
See General Pedagogic Standards below

GP4 **Knowledge of language that develops learners' mathematics.**
Foundation and Intermediate Phase teachers should be aware of the positive features and intentional use of code-switching and trans-languaging and know when and how to apply these. They must be mindful that learners will use prior linguistic schema, will learn when there is a need to communicate and to learn, and they will learn when they are motivated to do so.

GP5 **Knowledge of how to use appropriate practical and technological tools to support mathematical working.**

General pedagogic standards for mathematics teaching

GPM 1: Plan effective learning experiences

GPM 1.1 Developing learners' mathematics requires PPTs to know how to plan coherent learning experiences through the selection, sequencing and organisation of mathematics content. This knowledge sits within a broader awareness of trajectories and networks of mathematical ideas.

GPM 1.2 Plan coherent learning experiences by drawing on knowledge of children's mathematical learning and assessment of children's mathematical progress.

GPM 1.3 Make connections between various curriculum concepts and strategies to develop learner's mathematical understandings.

GPM 1.4 Select or develop tasks involving problem-solving and various problem-solving structures.

GPM 2: Take learners' knowledge into account

GPM 2.1 Design or select tasks and activities appropriate to learners' different levels of understanding and unique ways of thinking in order to push learners to more sophisticated levels of thinking.

GPM 2.2 Select and design tasks and examples that build on learner's prior knowledge.

GPM 2.3 Identify and address learner misconceptions and errors.

GPM 2.3 Understand the learning trajectories of learners in the primary school.

GPM 3: Engage learners productively with mathematics

GPM 3.1 Engage and develop learner's thinking through discourse, and communicate concepts, procedures and ideas through various modes of representation.

GPM 3.2 Incorporate the use of home languages when planning and delivering lessons.

GPM 3.3 Relate school mathematics to learners' out-of-school context integrating their home language.

GPM 3.4 Develop tasks to enhance learners' mathematical language that will assist them in improving their mathematical understandings.

GPM 4: Teach a balanced mathematics curriculum

GPM 4.1 Communicate concepts and procedures through various modes of representation.

GPM 4.2 Create opportunities for children to engage in the processes of problem-solving, conjecturing, reasoning, justifying and generalising.

GPM 4.3 Develop learners' computational fluencies and automaticity of key number facts and other mathematical fluencies.

GPM 4.4 Encourage learners to work flexibly and efficiently with different strategies when solving mathematical tasks.

GPM 4.5 Connect different representations to a single mathematical idea.

MKP Strand 1: Mathematical Acting and Thinking (MAT)

Purpose and Rationale

Doing mathematics is much more than reproducing symbols and procedures in a format specified by a teacher. It is about approaching the world and performing tasks in the world, in a way that is particularly mathematical. This mathematical approach requires two things:

- It requires mathematical content knowledge – the tools that we use to do mathematics.
- It requires mathematical thinking and action – ways of using these tools in our tasks and in the world that are flexible, creative and powerful.

Without mathematical content knowledge, mathematical thinking has no tools and no power. Without mathematical thinking, mathematical content knowledge is meaningless and sterile. This strand focuses on mathematical thinking and acting. The following strands focus on mathematical content knowledge. Both are needed for effective mathematics teaching.

Effective engagement in mathematical acting and thinking, requires learners to see themselves as doers of mathematics, not simply passive consumers. This orientation builds on the child's natural curiosity, playfulness, willingness to explore and desire to make sense of and act in their world.

Teachers need to facilitate their learners' engagement in mathematical practice so that they participate in the culture of authentic mathematical thinking and activity. To do this, teachers themselves need to have experienced the processes and practices that constitute authentic mathematical activity and support mathematical thinking. That is, teachers need to have the capacity to and experience of engaging effectively in mathematical thinking in appropriate contexts. Given the wide awareness of shortcomings in access to mathematics within basic education, it is imperative that time and resources are made available in pre-service primary teacher education for broad ranging access to what is involved in acting and thinking mathematically.

Characteristic Elements of the Mathematical Acting and Thinking

The particular standards and indicators of mathematical acting and thinking are detailed in terms of a number of identifiable characteristics of the mathematical process. These are dynamic elements that:

- indicate key elements of what is involved in Mathematical Acting and Thinking
- are never isolated, but always occur in conjunction with others
- occur across many different mathematical content areas.

The ordering and clustering of these standards provides some conceptual organization of these elements. But it **does not** imply any objective or temporal relationship between elements. That is:

- The ordering **does not** imply any developmental trajectory – learning to perform, or engage it, particular elements may occur in any order.
- The ordering **does not** imply any order of usage in the mathematical process – elements may be used in any order.

- The clustering **does not** imply that elements in a category will occur together – elements may occur in many different combinations in the mathematical process.

MAT Standard 1: Playful engagement to search for and develop mathematical insight
Engage in an open and free manner, to generate mathematical insight from situated and mathematical contexts or situations.

MAT 1.1 Act and imagine/visualise for insight

Seek mathematical insight by making and imagining physical transformations.

MAT 1.1.1 Relate tasks to concrete, perceptual and/or visualised objects.
MAT 1.1.2 Manipulate or transform these objects to generate mathematical insight.

MAT 1.2 Explore

Explore contextual and mathematical situations to generate relationships.

MAT 1.2.1 Describe observed relationships identified in patterns or processes.
MAT 1.2.2 Explain relationships with reference to underlying mathematical structures.

MAT 1.3 Connect:

Construct and formulate mathematical connections.

MAT 1.3.1 Seek and articulate relational, or analogous, connections between different mathematical and/or contextual situations
MAT 1.3.2 Seek and articulate connections between mathematical representations.

MAT 1.4 Clarify:

Pose and investigate questions to clarify understanding.

MAT 1.4.1 Identify areas of confusion or vague understanding.
MAT 1.4.2 Ask and interrogate questions to clarify and deepen understanding.

MAT Standard 2: Represent and use mathematics
Use mathematics to understand and regulate engagement in the world, and work with mathematical representations of many different forms to develop mathematical thinking and acting.

MAT 2.1 Identify attributes and invariants.

Identify properties in a situation that form countable or measurable attributes, or invariants.

MAT 2.1.1 Distinguish, identify, describe and compare, attributes that yield discrete (countable) or continuous (measurable) quantities.
MAT 2.1.2 Propose and use units (and scales) to measure attributes.
MAT 2.1.3 Identify invariants in situations or mathematical productions.

MAT 2.2 Describe and define

Mathematically describe and define.

MAT 2.2.1 Describe and define phenomena, using appropriate mathematical terms (and symbols).
MAT 2.2.2 Know the difference between defining (necessary and sufficient) properties and resultant properties.

MAT 2.3 Classify

Distinguish and organize objects to create mathematical structures.

MAT 2.3.1 Create categories by distinguishing mathematical objects based on similarities and differences
MAT 2.3.2 Organize these categories into mathematical structures.

MAT 2.4 Model

Analyse and represent situations using mathematical models.

MAT 2.4.1 Construct mathematical models, including standard mathematical models, of situations.
MAT 2.4.2 Pose mathematical questions about situations and associated models.
MAT 2.4.3 Interpret conclusions from models in the light of the situations.

MAT 2.5 Represent

Select, form and manipulate mathematical representations.

MAT 2.5.1 Work with a range of representations, including labels, names, diagrams, figures, symbol systems, and mappings (functions).
MAT 2.5.2 Form mathematical representations appropriate to the concepts, operations and relationships being investigated.
MAT 2.5.3 Compare strengths and weakness of different representations in relation to mathematical goals.

MAT Standard 3: Reason mathematically

Give and interrogate reasons for mathematical actions that are performed.

MAT 3.1 Specialize

Consider special cases to generate and check mathematical insight.

MAT 3.1.1 Simplify a task by focusing only on a part of it or on specific values.
MAT 3.1.2 Work with specific values to check general statements.

MAT 3.2 Generalise

Generalize patterns, relationships and attributes.

MAT 3.2.1 Formulate general forms of patterns, relationships, and attributes (inductive reasoning).

MAT 3.2.2 Explain patterns, relationships and attributes in terms of underlying mathematical structures or processes.

MAT 3.3 Conjecture

Generate and test conjectures.

MAT 3.3.1 Observe and make statements (conjectures) that open up possible relationships to be explored.
MAT 3.3.2 Generate cases to test the feasibility of conjectures.

MAT 3.4 Justify or Refute

Provide supporting reasons or refuting counter-examples.

MAT 3.4.1 Provide mathematically convincing reasons to justify decisions, processes and/or claims.
MAT 3.4.2 Construct counter-examples and show how they demonstrate that a conjecture is false.
MAT 3.4.3 Evaluate reasons to justify claims.

MAT 3.5 Prove

Validate conjectures.

MAT 3.5.1 Formulate sound and complete mathematical arguments to validate conjectures (deductive reasoning).
MAT 3.5.2 Evaluate the validity of arguments with reference to underlying premises (definitions).

MAT Standard 4: Reflect for action

Reflect on the process to make decisions for further action

MAT 4.1 Critique

Compare mathematical productions for efficiency, effectiveness and elegance.

MAT 4.1.1 Generate and use appropriate criteria to compare and judge different mathematical production for efficiency and effectiveness.
MAT 4.1.2 Discuss and debate the elegance of different mathematical productions.

MAT 4.2 Attend to precision

Use precision appropriate to a task.

MAT 4.2.1 Approximate quantities with precision appropriate to a task.
MAT 4.2.2 Use mathematical concepts and level of detail that are appropriate to the task

MAT 4.3 Regulate:

Reflect to regulate task process.

MAT 4.3.1 Reflect on task progress in order to regulate further engagement.

MKP Strand 2: Number and Algebra (NA)

Purpose and Rationale

Prospective primary teachers need to understand the different conceptions of number both from a cognitive science and from a mathematics education perspective. They need to be able to use this knowledge to draw on children's emergent quantitative awareness and initial understanding of number to facilitate the development of children's knowledge of number. In addition, they need to know the characteristics that children with number sense display and how to develop these characteristics.

Conception of number sense

Number Sense has both core biologically endowed (innate) features and culturally acquired (learnt, non-core) features. The innate features of number sense enable an early sensitising to quantity ('muchness') in human infants by means of the so-called Approximate Number System (ANS) and a system that enables humans to track small number of objects (OTS). Both systems remain operative over the lifespan of humans. The non-core features of Number Sense develop off the core, biologically endowed system through focused learning and teaching that exploit incipient arithmetic affordances of ANS and OTS. Well-developed Number Sense exhibits increased facility and flexibility in working with numbers and number relations.

NA Standard 1: Knowledge and use of the emergent number awareness of learners

Use and build on learners' initial quantitative awareness to develop their knowledge of number.

NA 1.1 Distinguish between types of quantity (discrete and continuous) and forms of quantification (counting and measure).

NA 1.2 Recognise learners' ability to subitize, approximate, compare and order numbers.

NA 1.3 Use one-to-one correspondence between sets to construct equivalence classes to which are assigned numbers, number names and numerals.

NA 1.4 Distinguish between lists of number words and lists of numerals, sequences of number words, sequences of numerals, and counting proper.

NA Standard 2: Knowledge of numbers systems

Use and build on learners' initial understanding of number to develop their knowledge of the number systems.

NA 2.1 Distinguish between place value (base 10 as well as other number bases) and non-place value number systems (and translate between different number systems).

NA 2.2 Describe the computational features of number systems (closure, commutativity, associativity, distributivity, inverses, identity and order).

NA 2.3 Explain the construction of the natural numbers.

NA 2.4 Explain how rational numbers, integers and real numbers are derived from the set of natural numbers.

NA Standard 3: Additive relations with natural numbers
Knowledge of the relationship between addition and subtraction.

NA 3.1 Show how counting is used to relate operations on sets to addition and subtraction.

NA 3.2 Identify and explore part-whole relationships and the link to addition and subtraction

NA 3.3 Demonstrate knowledge of equivalence relations in general and equality, in particular in relation to addition and subtraction.

NA 3.4 Describe and use addition and subtraction problem types and the range of associated representations.

NA 3.5 Identify learners' strategies used to solve addition and subtraction problems and introduce learners to more mathematically sophisticated strategies.

NA Standard 4: Multiplicative relations with natural numbers
Knowledge of the relationship between multiplication and division.

NA 4.1 Describe and use the relation between multiplication and division.

NA 4.2 Demonstrate knowledge of equivalence relations in general and equality in particular, in relation to multiplication and division problems.

NA 4.3 Demonstrate the necessity for the exclusion of division by zero.

NA 4.4 Demonstrate knowledge of number theory, in particular odd and even numbers, factorisation, prime factorisation, the divisibility rules and links to multiplication and division.

NA 4.5 Describe and use cardinal exponentiation as a foundation for teaching exponents and operations on numbers in exponential form.

NA 4.6 Describe and use multiplication and division problem types and the range of useful associated representations.

NA 4.7 Identify learners' strategies used to solve multiplication, division and exponentiation problems and introduce learners to more mathematically sophisticated strategies.

NA Standard 5: Rational numbers and operations on rational numbers
Knowledge of operations on rational numbers.

NA 5.1 Demonstrate how to teach learners to order, add, subtract, multiply and divide rational numbers.

NA 5.2 Explain how multiplicative reasoning and proportional reasoning relate to fractions, decimals, percentages, ratios, proportions and rates.

NA 5.3 Identify and use part-whole relationships when working with fractions, decimals, percentages, ratios, proportions and rates.

NA 5.4 Describe and use a range of representations, calculation strategies and tasks that are used to solve problems involving rational numbers.

NA Standard 6: Integers and operations on integers
Knowledge of operations on integers.

NA 6.1 Demonstrate how to teach learners to order, add, subtract, multiply and divide integers.

NA 6.2 Describe and use a range of representations, calculation strategies and tasks that are used to solve problems involving integers.

NA Standard 7: Early algebra
The use of algebraic reasoning in arithmetic and algebra.

NA 7.1 Use algebraic reasoning to geneneralise and justify the basic arithmetic operations on numbers.

NA 7.2 Distinguish between inductive and deductive reasoning with respect to the use of sequences, series and patterns.

NA 7.3 Demonstrate how patterns, sequences and series can be treated as functions.

NA 7.4 Use equivalence and equality in the transformation of algebraic expressions and equations.

NA Standard 8 Overarching mathematical ideas for school arithmetic
Understand and apply overarching mathematical ideas to school arithmetic

NA 8.1 Apply knowledge of relations and functions in order to understand the nature of mathematical operations.

NA 8.2 Apply knowledge of equivalence relations, equivalence classes, equality, equations and order relations.

NA 8.3 Explain and identify sets, set membership, set relations, set operations, and set partitions.

NA 8.4 Explain parthood and the different ways in which quantities can be viewed in terms of part-whole relations and constructed through part-whole operations.

NA 8.5 Use structure preservation to understand the relation between part-whole operations and operations on numbers; and the relation between set operations and operations on numbers.

NA 8.6 Use structure preservation to understand the relation between manipulatives and models and basic arithmetic operations.

MKP Strand 3: Geometry and Measurement (GM)

Purpose and Rationale

In the teaching of Geometry and Measurement prospective teachers are expected to know and be able to teach three content standards.

GM 1 Knowledge of Geometrical Properties

The properties of geometric elements make those geometric elements what they are. Initial properties are most likely to be those attributes which are initially perceived when any geometric element is observed.

GM 1.1 Describe lived space and objects use of the minimum information required to locate a point in space.

GM 1.2 Explain 0-D (point), 1-D (line), 2-D (shape), and 3-D (object)

GM 1.3 Describe the location of the point

GM 1.4 Understand that lines can be defined in terms of their position, direction, and in relation to other lines

GM 1.5 Know that angles are formed when lines intersect

GM 1.6 Realise that 3-D objects can consist of flat and/or curved surfaces, and 2-D shapes can consist of straight and/or curved lines

GM 1.7 Realise the 3-D objects and 2-D shapes can be deconstructed into other shapes and objects

GM 1.8 Know that 3-D objects and 2-D shapes can be described, classified and named according to their properties

GM 1.9 Know that polyhedrons and polygons can be constructed from, and deconstructed into points, lines, and (other) polygons

GM 1.10 Understand that some polyhedrons and polygons possess the properties of other polyhedrons and polygons, and can therefore be classified and named in more than one way

GM 1.11 Accept that polyhedrons and polygons can be accurately defined using necessary and sufficient properties

GM 2 Knowledge of Measurement

Understand that measurement is a number that indicates a comparison between the attribute of an object being measured and the same attribute of a given unit of measure.

GM 2.1 Recognise the attribute being measured

GM 2.1.1 Know that the extent of an attribute is the extent of the measurement.

GM 2.1.2 Identify the (measurable) attribute of the object being measured.

GM 2.2 Identify a suitable unit

GM 2.2.1 Select a standard unit that correlates (dimensionally) with the attribute being measured.

GM 2.3 Recognise the cardinality of the units employed

GM 2.3.1 Know that the total number of units constituting the extent being measured establishes the final measure.

GM 2.4 Iterating units

GM 2.4.1 Explain that a measure is constituted through iterating the selected unit across the extent of the attribute of the object being measured.

GM 2.5 Estimating

GM 2.5.1 Use estimation as a means of demonstrating an understanding of units and the measurement process.

GM 2.6 Integrating knowledge of number with knowledge of measurement

GM 2.6.1 Use measurement to mediate relations between real-world contexts and number knowledge when quantifying objects encountered in the environment.

GM 2.7 Understand relations between different applications of measurement (i.e. length, area, volume, capacity, time, mass, etc.)

GM 2.7.1 Use connections between length, area, volume, capacity, mass, time and other measures to develop a coherent conception of measurement

GM 3 Knowledge of Transformations

Transforming geometric objects involves changing their position (translation), orientation (rotation, reflection), size (dilation), and/or shape (deformation). This involves five processes, taken in isolation or in some or other combination:

GM 3.1 Understand, represent and construct transformations

GM 3.1.1 Identify, construct and represent translations, reflections, rotations, dilations, and deformations, of objects in space.

GM 3.1.2 Identify, construct and represent translations, reflections, rotations, dilations, and deformations, of shapes in a plane.

GM 3.2 Work with geometric patterns

GM 3.2.1 Identify similarities and differences between geometric patterns (tessellations) that share common characteristics (e.g. form, line, symmetry, angle, and vertex arrangement, space).

GM 3.2.2 Construct representations of transformations of 2-D geometric shapes and/or 3-D objects using tessellations.

GM 3.3 Solve spatial visual problems

GM 3.3.1 Use properties of objects and shapes in relation to the principles of transformations to solve spatial problems.

Recommendation: BEd credits for mathematics

The PrimTEd assessment workstream estimates that to meet these standards approximately 100 credits are needed across the 480 credit B.Ed degree for students specializing in the Foundation or the Intermediate Phases.